International Journal of Applied Mathematics & Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 4, Issue 4, Jun - Jul 2015, 15-24

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

OPTIMIZATION OF REAL TIME DISRUPTION MANAGEMENT FOR A TWO-STAGE BATCH PRODUCTION – FUZZY INVENTORY SYSTEMS WITH RELIABILITY CONSIDERATIONS VIA GEOMETRIC PROGRAMMING

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ABSTRACT

The purpose of this paper is to discuss the fuzzy EOQ model for a two-stage production – inventory system with reliability considerations. In this paper we establish and analyse two Economic order quantities (EOQ) that in this model, some parameters are fuzzy variables. This note is based on inventory models under total cost minimization and profit maximization that have solved via fuzzy geometric programming. (FGP) techniques by A. Kordi (2010). Through FGP and by Zadeh's extension Principle, two main Programs are transformed to a pair of two-level of mathematical programs. The upper bound and lower bound of the objective value are obtained by solving the pair of geometric programs.

KEYWORDS: Inventory, Fuzzy Geometric Programming, EOQ

1. INTRODUCTION

Batch Production System are very common in manufacturing industries and these industries, depending on the type of products, both single and multiple, Products may be produced using the same production facility.

In single stage production inventory systems, include a single-item inventory systems with non-stationary demand process, determination of lot size and order level for a Single-item inventory model with a deterministic time-dependent demand, a Single-item periodic review stochastic inventory system and a Single-item Single-Stage inventory system with stochastic demand with periodic review where the system must order either none or atleast as much as a minimum order quantity.

The studies in two-stage production systems include optimal lot sizing or minimize the sum of all costs, Steady – State average inventory and an effective production ordering policy in a capacity – constrained production and inventory system and a direct and intuitive way of deriving the lot sizes.

In this paper, two fuzzy inventory models which extend the classical economic order quantity (EOQ) model are developed and analysed. One model uses total cost minimization of the other model used profit maximization. In each models, order quantity (Q) is a fuzzy variable Jung and Klein [11] developed the crisp model, namely Q was a crisp variable, but in this note we assume that Q is a fuzzy variable and for deriving and analyzing the optimal solution, We employ Fuzzy Geometric programming (FGP) technique. There are two main factors for developing these two models. The first factor is the use of FGP to derive the optimal solution. The FGP approach can be effectively applied to this model, a pair of two level mathematical program is formulated to calculate the upper and lower bounds of the objective value at possibility level by Shiang Tailiu.

The membership function of the fuzzy objective value is derived numerically by enumerating different values. The remainder of the paper is organized as follows. In section 2, we give a brief overview for the basic building blocks namely, fuzzy set theory and real time disruption management for a two-stage batch production inventory system with reliability considerations. Section 3 and section 4 develops the fuzzy model with the total cost minimization and profit maximization. Section 5 provides a detailed numerical Illustration along with a discussion of key insights. Section 6 concludes the paper.

2. PRELIMINARIES

2.1 Definition: Fuzzy Set

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0,1]\}$. In the pair $(x, \mu_{\tilde{A}}(x))$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0, 1], called membership function or grade of membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

2.2. α - Cut

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α level set or α - cut. $A(\alpha) = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$.

2.3 Generalized Fuzzy Number

Any fuzzy subset of the real lien R, whose membership function satisfies the following conditions, is a generalized fuzzy number

i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1].

ii) $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq a_1$,

iii) $\mu_{\tilde{A}}(x)$ = L(x) is strictly increasing on $[a_1, a_2]$

iv) $\mu_{\tilde{A}}(x) = 1$, $a_2 \leq x \leq a_3$,

v) $\mu_{\tilde{A}}(x)$ = R(x) is strictly decreasing on $[a_3, a_4]$

vi) $\mu_{\tilde{A}}(x) = 0$, $a_4 \leq x < \infty$, where a_1, a_2, a_3 and a_4 are real numbers.

2.4 Trapezoidal Fuzzy Number

The fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, where $a_1 < a_2 < a_3 < a_4$ and defined on R is called the trapezoidal fuzzy number, If the membership function $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & : x < a_1 \text{ or } x > a_4 \\ \dfrac{(x - a_1)}{(a_2 - a_1)} & : a_1 \leq x < a_2 \\ 1 & : a_2 \leq x < a_3 \\ \dfrac{(x - a_4)}{(a_3 - a_4)} & : a_3 \leq x \leq a_4 \end{cases}$$

2.5 Assumptions and Notations

Assumptions:

1. The production rate of the item is greater than its demand rate.
2. The original production line is perfectly balanced, that means the production rates for the item are equal for the both stages.
3. The recovery Cycle will start just after the disruption occurs. The disruption can occur in either single or both stages at any point in time.
4. There are equal numbers of cycles in the disruption recovery windows of both stages.
5. All products are inspected and defective products are rejected
6. The total cost of interest and depreciation per production cycle C (A, r) is inversely related to Set-up cost (A) and is directly related to process reliability (r) according to the following general power function.

   $C(A,r) = aA^{-b}r^{c}$

   Where a, b and c are positive constant chosen to provide the best fit of the estimated cost function.

Notations:

D $\rightarrow$ demand per year (Unit Per year)

$H_1 \rightarrow$ Holding cost per unit per year at the first stage ($/Unit/Year)

$H_2 \rightarrow$ Holding cost per unit per year at the second stage ($/Unit/Year)

r $\rightarrow$ Reliability of the production process – which is known from the historical

date of the production system.

Q $\rightarrow$ Combined production lot size per normal cycle with reliability r

$A_1 \rightarrow$ Set-up cost per cycle at the first stage ($ per Set-up)

$A_2 \rightarrow$ Set-up cost per cycle at the second stage ($ per Set-up)

P $\rightarrow$ Production rate (Units per year) in 100% reliable system

The annual cost of the set-up cost and the holding cost which is

$$TC(Q) = \frac{DA_1 + DA_2}{Q} + \frac{Q}{2}\left(\frac{H_1 D}{rP} + \frac{H_2 D}{rP}\right)$$

By minimizing the total cost, we obtain the optimal value

$$Q = \sqrt{\frac{2rP(A_1 + A_2)}{H_1 + H_2}}$$

2.6 Geometric Programming

Geometric Programming has been very popular in engineering design research since its inception in the early 1960s. Even though GP is an excellent method to solve nonlinear problems, the use of GP in inventory models has been relatively in frequent. Kochenberger [12] was the first to solve the basic. EOQ models using GP.

Consider

$P_1 = Q^1$; $P_2 = Q^{-1}$

$a_{11} = 1$; $a_{12} = -1$

$y_1 + y_2 = 1$

$$\sum_{j=1}^{2} a_{ij} = O$$

$$a_{11}y_1 + a_{12}y_2 = 0 \therefore y_1 - y_2 = O$$

$$\therefore y_1 = y_2 = \frac{1}{2}$$

$$TC(Q) = \left(\frac{D(H_1+H_2)}{2rPy_1}\right)^{y_1}\left(\frac{D(A_1+A_2)}{y_2}\right)^{y_2}$$

$$= \left[\frac{D(H_1+H_2)}{2rP\frac{1}{2}}\right]^{\frac{1}{2}}\left[\frac{D(A_1+A_2)}{\frac{1}{2}}\right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{2D^2(H_1+H_2)(A_1+A_2)}{rP}}$$

$$y_1 TC(Q) = \frac{1}{2}\left(\frac{D(H_1+H_2)}{rP}\right)P_1$$

$$\frac{1}{2}\sqrt{\frac{2D^2(H_1+H_2)(A_1+A_2)}{rP}} = \frac{1}{2}\left(\frac{D(H_1+H_2)}{rP}\right)Q$$

$$\frac{2(H_1+H_2)(A_1+A_2)}{rP} = \frac{(H_1+H_2)^2}{r^2P^2}Q^2$$

$$\therefore Q = \sqrt{\frac{2rP(A_1+A_2)}{H_1+H_2}}$$

3. MINIMIZING MODEL

We have the following mathematical formulation for the total cost per unit time (TC) with order quantity Q becomes the following FGP.

$$\tilde{TC}(Q) = \tilde{D}(A_1+A_2)Q^{-1} + \frac{Q}{2}\left(\frac{\tilde{D}(\tilde{H}_1+\tilde{H}_2)}{r}\right)P^{-1}$$

Where Q is the decision variable and Q would be determined.

3.1 Optimal Solution Procedure

According to Fuzzy Geometric Programming Objective function in this model is an Unconstrained Polynomial with Zero degree of difficulty by Guardiola (2009). So by above definition and assumption, we are facing by this mathematical formulation for the total fuzzy cost per unit time from the FGP perspective.

$$\tilde{TC}(Q) = \tilde{D}(A_1 + A_2)Q^{-1} + 0.5Q\left[\frac{\tilde{D}(\tilde{H}_1 + \tilde{H}_2)}{r}\right]P^{-1}$$

The objective function in this model is a fuzzy unconstrained polynomial with zero degree of difficulty. In this part for easily solving, we use the duality FGP techniques. Suppose that $\tilde{H}$ *and* $\tilde{D}$ are trapezoidal fuzzy number

$$S = \left\{(\tilde{H}, \tilde{D}) \mid (H_1)_\alpha^L, (H_2)_\alpha^L, (\tilde{H})_\alpha, (H_1)_\alpha^U, (H_2)_\alpha^U, (D)_\alpha^L, \tilde{D}, (D)_\alpha^U\right\}$$

for each $TC(\tilde{H}, \tilde{D}) \in S$

We denote $TC(\tilde{H}, \tilde{D})$ to be the objective value of this model. Let TC^L and TC^U be the minimum and maximum of $TC(\tilde{H}, \tilde{D})$ on S respectively, namely

$$Z^L = \min\left\{TC(\tilde{H}, \tilde{D}) \mid (\tilde{H}_1, \tilde{H}_2, \tilde{D}) \in S\right\} \text{ and}$$

$$Z^U = \max\left\{TC(\tilde{H}, \tilde{D}) \mid (\tilde{H}_1, \tilde{H}_2, \tilde{D}) \in S\right\}$$

Which can be reformulated as the following pair of two-level mathematical program.

$$TC^L = \min_{(\tilde{H}, \tilde{D}) \in S} \min \tilde{D}(A_1 + A_2)Q^{-1} + 0.5Q\left[\frac{\tilde{D}(\tilde{H}_1 + \tilde{H}_2)P^{-1}}{r}\right]$$

$$TC^U = \max_{(\tilde{H}, \tilde{D}) \in S} \max \tilde{D}(A_1 + A_2)Q^{-1} + 0.5Q\left[\frac{\tilde{D}(\tilde{H}_1 + \tilde{H}_2)P^{-1}}{r}\right]$$

For solving we use dual based algorithms so one can transform models to the corresponding dual geometric program as follows.

$$Tc^L = \max_y \left(\frac{(D)_\alpha^L (A_1 + A_2)}{y_1}\right)^{y_1}\left(\frac{0.5\left((H_1)_\alpha + (H_2)_\alpha\right)^U DP^{-1}}{r\,y_2}\right)^{y_2}$$

Subject to $y_1 + y_2 = 1$

$-y_1 + y_2 = 0$

$y_1, y_2 > 0$

$$TC^L = \max_{(H, D) \in S} \max \left[\frac{D\left(A_1 + A_2\right)_L}{y_1}\right]^{y_1} \left[\frac{0.5\left((H_1) + (H_2)\right)_U DP^{-1}}{r\, y_2}\right]^{y_2}$$

Subject to $y_1 + y_1 = 0$

$-y_1 + y_2 = 0$

$y_1, y_2 > 0$

If we define the i^th^ term of the optimal primal objective function as U_i^*

$$U_1^* = D_\alpha \left(A_1 + A_2\right)_\alpha^L \left(Q^L\right)^{-1}$$

$$U_2^* = 0.5\left\{\left(H_1\right)_\alpha^U + \left(H_2\right)_\alpha^U\right\} D P^{-1}\, Q^L \big/ r$$

The optimal order quality can be calculated from above as follows.

$$Q^L = \sqrt{\frac{2rP\left(A_1 + A_2\right)}{\left(H_1\right)_\alpha^U + \left(H_2\right)_\alpha^U}}$$

$$Q^U = \sqrt{\frac{2rP\left(A_1 + A_2\right)}{\left(H_1\right)_\alpha^L + \left(H_2\right)_\alpha^L}}$$

4. MAXIMIZATION MODEL

We are facing formulation for the profit per unit time.

$$Max\, \pi(Q, D) = D\left(A_1 + A_2\right) Q^{-1} + \left\{0.5\left(H_1 + H_2\right)^{-\beta}(D) P^{-1}\right\} \Big/ r + AD^{1-\alpha} - BD^{1-\beta}$$

Where Q and D are decision variables.

$S = \left\{H \mid \left(H_1\right)_\alpha^L + \left(H_2\right)_\alpha^L \le H \le \left(H_1\right)_\alpha^U + \left(H_2\right)_\alpha^U\right\}$ for each $H \in S$, we denote $\pi(H)$ to be the objective value of this model.

$$\pi^L = \min\left\{\pi(H) \mid H_1, H_2 \in S\right\}$$

$$\pi^U = \max\left\{\pi(H) \mid H_1, H_2 \in S\right\}$$

which can be reformulation as the following pair of 2 level mathematical programming.

$$\pi^L = \min_{\tilde H \in S} \min_{Q,\tilde D} \left\{ A^{-1} D^{\alpha-1} + A^{-1}\left(A_1 + A_2\right) D^{\alpha} Q^{-1} + A^{-1} B D^{\alpha-\beta} + 0.5 A^{-1} D^{\alpha-\beta-1}\left(\tilde H_1 + \tilde H_2\right) P^{-1} \right\} / r$$

$$\pi^U = \max_{\tilde H \in S} \max_{Q,\tilde D} \left\{ A^{-1} D^{\alpha-1} + A^{-1}\left(A_1 + A_2\right) D^{\alpha} Q^{-1} + A^{-1} B D^{\alpha-\beta} + 0.5 A^{-1} D^{\alpha-\beta-1}\left(\tilde H_1 + \tilde H_2\right) P^{-1} \right\} / r$$

For two model π^L and π^U, we use the dual problem which is usually easier to solve, first for π^L

$$Max\, D = \left(\frac{1}{y_0}\right)^{y_0} \left(\frac{A^{-1}\lambda}{y_1}\right)^{y_1} \left(\frac{A^{-1}\left(A_1 + A_2\right)\lambda}{y_2}\right)^{y_2} \left(\frac{A^{-1}B\lambda}{y_3}\right)^{y_3} \left(\frac{0.5 A^{-1} P^{-1}\left(\tilde H\right)_{\alpha}^{L}\lambda}{r y_4}\right)^{y_4}$$

Subject to $y_o = 1$

$-y_0 + y_1 = 0$

$$(\alpha - 1) y_1 + \alpha y_2 + (\alpha - \beta) y_3 + (\alpha - \beta - 1) y_4 = 0$$

$$-y_2 + y_4 = 0$$

$$y_i > 0,\ i = 1, 2, 3, 4$$

$$where\ \lambda = y_1 + y_2 + y_3 + y_4$$

According to the dual techniques

$$Q = \left(\frac{A_1 + A_2}{A V_2}\right)\left(\frac{A V_3}{B}\right)^{\frac{1}{\alpha-\beta}}$$

$$D_{\alpha} = \left(\frac{A V_3}{B}\right)^{\frac{1}{\alpha-\beta}}$$

Such that

$$V_2 = A^{-1}\left(A_1 + A_2\right) D^{\alpha} Q^{-1}$$

$$V_3 = P^{-1} A^{-1} B D^{\alpha-\beta}$$

5. NUMERICAL EXAMPLE

Let D = 400,000, r = 0.90, H_1=1.2, H_2=1.3

P = 500,000, A_1=50, A_2=30

Q = 5366.561

TC (Q) = 11925.6958

Let demand per unit time be a trapezoidal number

$$\tilde H_1 = (0.8, 1, 1.4, 1.6)$$

$$\tilde H_2 = (0.9, 1.1, 1.5, 1.7)$$

In this model for minimization of cost, we’ve

$$\tilde{H}_1 = \left[0.8 + 0.2\alpha,\ 1.6 - 0.2\alpha\right]$$

$$\tilde{H}_2 = \left[0.9 + 0.2\alpha,\ 1.7 - 0.2\alpha\right]$$

$$Q^L = \sqrt{\frac{2rP\left(\left(A\right)_U + \left(A\right)_U\right)}{\left(H_1\right)_\alpha + \left(H_2\right)_\alpha}}$$

$$Q^L = \sqrt{\frac{2 \times 0.9 \times 5{,}00{,}000 \times 80}{\left(1.6 - 0.2\alpha\right) + \left(1.7 - 0.2\alpha\right)}}$$

$$Q^U = \sqrt{\frac{2rP\left(\left(A\right)_L + \left(A\right)_L\right)}{\left(H_1\right)_\alpha + \left(H_1\right)_\alpha}}$$

$$Q^U = \sqrt{\frac{2 \times 0.9 \times 5{,}00{,}000 \times 80}{\left(0.8 + 0.2\alpha\right) + \left(0.9 + 0.2\alpha\right)}}$$

6. CONCLUSIONS

In this paper, we developed and analyzed two EOQ based inventory models under total cost minimization and profit maximization and profit maximization the some parameters are fuzzy. By using FGP techniques, we determined the order quantity or the total cost minimization and profit maximization model to transform for main program in two models in to two level geometric programs.

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